Ultra-Tight GPS/INS/PL Integration: Kalman Filter Performance Analysis

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Abstract: The smooth functioning of the integration Kalman filter is vital to the performance of an ultra-tight integration system, as the integration filter not only provides error estimates of the inertial sensor measurements but also provides a Doppler feedback to the receiver tracking loops to mitigate dynamics on GPS signals. The I (in-phase) and Q (quadrature) measurements from the correlator which forms the input to the filter are highly non-linear, and therefore need to be linearised during the measurement update process. From the derivations of I and Q signals, it can be seen that these quadrature measurements are related to phase and frequency errors. Therefore, linearisation of these quadrature signals is carried out with respect to phase and frequency errors.

The performance of the Kalman filter is primarily driven by the quality of the modelling strategies. Any mis-modelling, either due to lack of statistical knowledge of the signal characteristics or due to improper assumptions, will cause the filter to diverge. Given the fact that the signals are highly non-linear, modelling plays an important role in the design of the Kalman filter. The number of states chosen for the filter is a trade-off between optimal performance and computational complexity. Our analysis shows that a 17-state filter would suffice. Covariance analysis is performed to test the performance of the filter under various operating conditions. In this paper, emphasis is placed on measurement update, which relates the measurements and states, as it is a unique characteristics compared with the loose and tight integration modes. This paper also describes the pertinent mathematical relationships that are required to develop the measurement model. Various trajectories with different dynamics are studied to evaluate the modelling effects.

Keywords. Ultra-tight Kalman filter architecture, I, Q measurements, states-measurements relationship

1 Introduction

The complementary advantages of Global Positioning System (GPS)/Pseudolites (PL) and Inertial Navigation System (INS) sensors overcoming each other's limitations have been the primary motivation for the integration of these systems. Since their inception in the early 1980's, the GPS/INS integrated system has undergone major changes in integration architectures, algorithms, real-time implementations, hardware/software, etc. Market studies reveal that the major revenues for Satellite Based Navigation Systems, such as GPS, GLONASS and the upcoming GALILEO, will be primarily from the commercial sector (Rizos, 2005). This is due to the proliferation of applications such as LBS (Location Based Services) and Telematics. Though Network Based Positioning Techniques can augment Satellite Based Systems, nevertheless the infrastructure requirements for this are quite high. Therefore low-cost INSs are seen as an alternative which can augmentSatellite Based Systems to provide robust positioning (Titterton & Weston, 1997). In addition, the improved performance of the presently used GPS/INS architectures in non-benign environments have popularised their use in both commercial and defence applications.

Considering GPS and INS as two independent navigation systems and integrating their positions externally in a Kalman filter is defined as the loosely-coupled mode. Two Kalman filters are used in a cascaded fashion in this type of system – a navigation filter inside the GPS receiver, and the integration Kalman filter which combines both the GPS and INS outputs. Usually, the covariance knowledge of the navigation filter is not provided to the external integration Kalman filter. This lack of knowledge results in sub-optimal performance. The other disadvantage of such a system is that as GPS is treated as a navigation system, a minimum of 4 satellites should be tracked by the receiver to provide the coordinate/velocity inputs to the integration Kalman filter. To improve upon this, the tight integration mode was developed,

which combines INS data with GPS pseudo-ranges or carrier phases (Sennott, 1997; Sennott, 1999). The biggest advantage of this system is that it uses only one Kalman filter, which improves the system performance greatly. Moreover, GPS is treated as a sensor rather than a closed navigation system, and therefore this system can even provide navigation outputs even with less than 4 tracked satellites (Brown & Hwang, 1997). But, as navigation systems are increasingly subjected to non-benign environments where higher performance is required, designers have conceived of the ultra-tight integration mode. In such a system, the GPS measurements I (in-phase) and Q (quadrature) from the GPS correlator are integrated with the INS measurements (Alban et. al, 2003; Kreye et. al, 2002; Poh et. al, 2000; Kim et. al, 2003). What make this mode more attractive than the previous two architectures are the manifold advantages it can provide, such as Jamming to Signal (J/S) ratio improvement, mitigating RF interference, improving GPS measurement accuracy, reducing non-coherent integration period in weak-signal GPS processing and others. These advantages, in addition to the increasing demands of critical applications, have made such system architecture attractive.

An ultra-tight system derives its benefits primarily from the INS-derived Doppler feedback to the receiver carrier tracking loops. This derived Doppler signal, which closely reflects the Doppler (caused due to relative motion between satellite and receiver) on the GPS signals, when integrated with the tracking loop removes the Doppler from the GPS signals, thereby facilitating a significant reduction in the carrier tracking bandwidth, i.e. from about 12 to 18Hz to about 1 to 3Hz depending on the oscillator's accuracy. Unlike a stand-alone GPS receiver where individual channels are controlled within the correlator, in an ultra-tight integrated system the loops are closed by the integration Kalman filter (Beser et. al, 2003). A centralised Kalman filter or a federated Kalman filter structure can be adopted for integration. Though having a single Kalman filter reduces the modelling complexity, nevertheless, the higher update requirements make this unattractive for real-time implementation.

The measurement update of the Kalman filter requires a relationship to be established between the states and measurements. Therefore, in an ultra-tightly integrated system the relationship between GPS measurements, I

and Q, and INS data, position, velocity and attitude, needs to be established. It is not as straightforward as in the case of loosely and tightly integrated systems. This paper shows that they are related through phase and frequency errors which are extracted from the tracking loops of the receiver. The implementation of these mathematical relationships in real-time in addition to two levels of synchronisation, one at the integration level and the other at Doppler synchronisation at the tracking loops, makes this system complex. Normally, the Kalman filter update rate is 1 to 10Hz, whereas in an ultra-tight system as the measurements from the correlator come at rates of 1000Hz the implementation of Kalman filter can be done in two ways – run the Kalman filter at the same rate as the measurements which demands high computational power or decimate the measurements to lower rates. For our simulation experiments, a 17-state Kalman filter is chosen which runs at 100Hz. A Software GPS receiver is used to generate the I and Q measurements, and an INS Matlab toolbox is used to generate the inertial measurements. A dynamic trajectory is chosen to test the performance of the ultra-tight Kalman filter and the results show good performance.

2 Ultra-tight Integration Measurements

The GPS correlator processes the digitised IF signals to generate the 50Hz navigation data. There are two loops that work in tandem to do this process – the carrier loop which locks on to the incoming carrier frequency and measures the apparent Doppler shift of the signal, either using a FLL or PLL or a combination of both, and the code loop which normally is a DLL (that compares the incoming spreading code with three versions of a locally generated code to determine the correlation peak, (Tsui, 2000; Ward, 1998). The functions performed by both these loops are generally referred to as Carrier or Doppler wipe-off and code wipe-off. Mixing the incoming composite signal with the locally generated signals and integrating them over the pre-detection interval period yields the I and Q measurements. These measurements are used in the discriminator algorithms to generate the corrections to the carrier and code NCO's to align to the incoming signal.
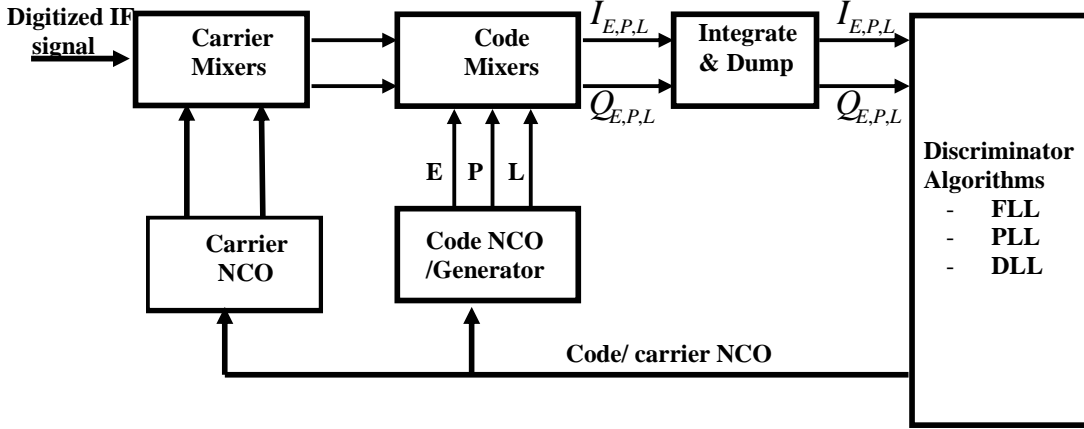
Fig. 1 Correlator Architecture

Figure 1 shows the architecture of a typical correlator used in a GPS receiver. For a detailed explanation the reader is referred to (Kaplan, 1996). The I and Q signals generated by the carrier mixers are converted to $I_{E,P,L}$ (in-phase early, prompt and late) and $Q_{E,P,L}$ (quadrature early, prompt and late) by the code mixers, which are subsequently integrated over the pre-detection interval. These signals are then processed by the various discriminator algorithms of the FLL, PLL and DLL to generate the code and carrier NCO corrections.

2.1 Kalman Filter Measurements

In a conventional receiver, the function of the I and Q signals is to determine the NCO corrections, and to compute the signal power $\sqrt{I^2 + Q^2}$ to determine in which loop, i.e. FLL, PLL, wideband / narrowband DLL, the receiver should operate. But, in ultra-tight integration these measurements also calibrate the inertial sensor errors once the relationship with position, velocity is established. In the steady-state condition, the measurements I_P (in-phase early) and Q_P (quadrature early) become the input to the integration Kalman filter. Let $\hat{w}$ and $\hat{\phi}$ be the local estimates of the receiver, k be the measurement epoch, and T be the integration interval. As mentioned in the previous section, multiplying the local carrier estimate with the incoming signal and integrating across the pre-detection interval yields the quadrature signals:

$$I = \int_{kT}^{(k+1)T} \sin(\hat{w}t + \hat{\phi})\left[A\cos\left(w't + \phi'\right) + \eta\right] dt$$

(1a)

$$Q = \int_{kT}^{(k+1)T} \cos(\hat{w}t + \hat{\phi})\left[A\cos\left(w't + \phi'\right) + \eta\right] dt$$

(1b)

Expanding these equations and averaging them over the integration period yields (Sennott, 1992):

$$E[I] = \frac{-A}{2w_e}\left[\cos(w_e(k+1)T + \varphi_e - \cos(w_e kT + \varphi_e)\right]$$

(2a)

$$E[Q] = \frac{A}{2w_e}\left[\sin(w_e(k+1)T + \varphi_e - \sin(w_e kT + \varphi_e)\right]$$

(2b)

where $E[I]$ and $E[Q]$ are the expectations of the I and Q measurements respectively, A is the amplitude of the signal, w_e is the frequency error between the measured and estimated signals, φ_e is the phase error between the measured and estimated signals. The I and Q signals of the four different PRN codes, 4, 6, 7, 10, which are used as input to the Kalman filter, are simulated as shown in Figure 2.

Fig. 2 Kalman filter measurements for Ultra-tight integration
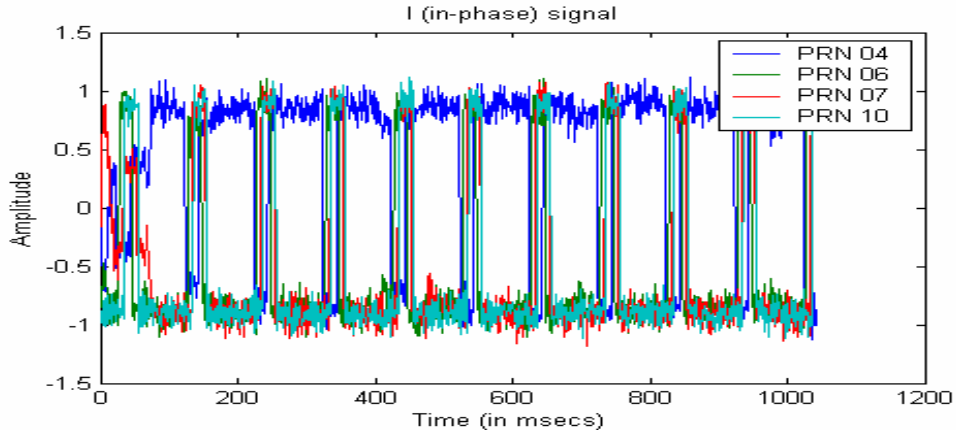
(a)   I (in-phase) signal from Correlator
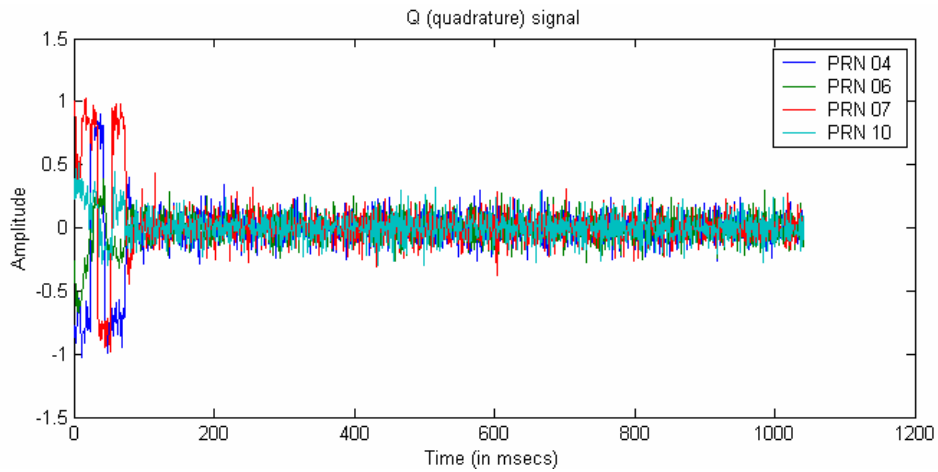
(b)   Q (quadrature) signal from Correlator

2.1 States – Measurement Relationship

Both the code and carrier loops need to be synchronised with the incoming signal to produce the navigation data.   A threshold based on the dynamics and signal-to-noise ratios is set to determine if the loops are locked or not.   Under normal conditions the loops operate in the steady-state condition, indicating that the errors generated by the loops are within the limits.   However, under high dynamic conditions, such as high accelerations and jerks, the loops may not be able to track the incoming signal due to the transients (Cahn et. al, 1977; Jwo, 2001).   In other words, the loop filter's bandwidth may not be sufficient to be able to track the sudden changes in the Doppler frequency.   To reduce the phase and frequency errors and maintain the loops in lock, generally, there are

two options – increase the tracking bandwidth (which will degrade the raw measurement accuracy), or aid the tracking loops using external signals/measurements from an INS. INS-aiding is optimal in that it reduces the dynamic stress, and at the same time improving the accuracy of the measurements.

In this integration technique, the phase and frequency errors are the variables that establish the relationship between the states and measurements in the integration Kalman filter. In equation, $w_e = \hat{w} - w'\, and\, \varphi_e = \hat{\varphi} - \varphi'$ are the frequencies and phase errors respectively. These errors are initially determined from the apriori knowledge but eventually reach a steady-state.
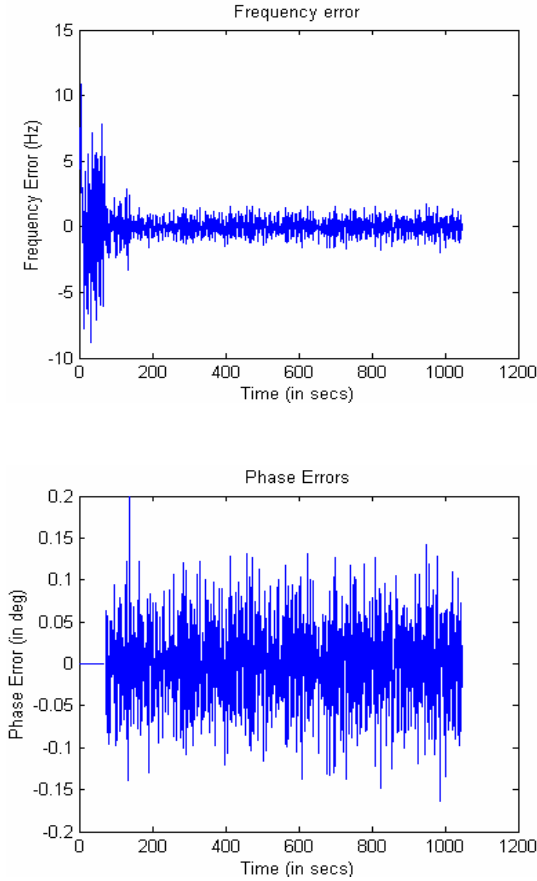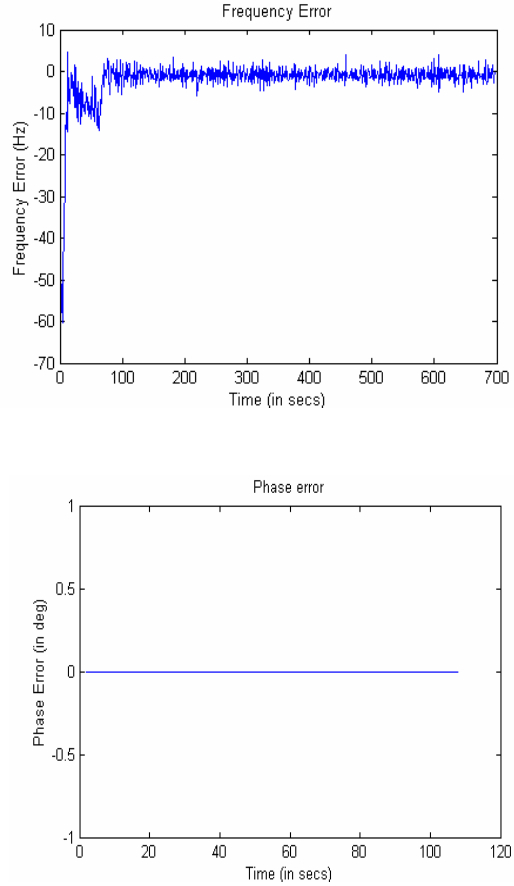
Figures 3 and 4 show the frequency and phase errors for both the ultra-tight and stand-alone systems. It can be observed that while the ultra-tightly integrated system can track the signals with a bandwidth of 3Hz, the stand-alone receiver could not even track the signals with bandwidth of 13Hz, and the loops remained in the FLL mode and never entered the PLL mode. These frequency and phase errors, which are the *'linking'* variables between the correlator measurements I and Q, and INS variables, position, velocity, are defined as:

$$w_e = \frac{w}{c} V_e \tag{3}$$

$$\varphi_e = \frac{-w}{c} \left[X_e - V_e t \right] \tag{4}$$

where $w = 2\pi f$ is the angular frequency, c is the velocity of light, V_e is the velocity error vector between the measured and estimated values, and X_e is the position error vector between the measured and estimated values. Under steady-state tracking conditions, i.e. when $w_e, \phi_e \leq threshold$, the magnitude of I increases and the magnitude of Q decreases. Having defined the relationships between the I and Q, phase and frequency errors, position, velocity, it is now straightforward to relate these using the following expressions:

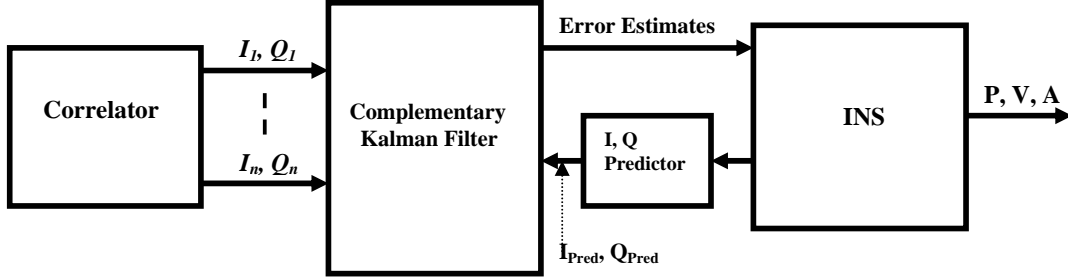$$dI = \left[\{\partial I, \partial \phi_e\}_{dx} + \{\partial I, \partial w_e\}_{dx} \right] dx \tag{5}$$

$$dQ = \left[\{\partial Q, \partial \phi_e\}_{d\dot{x}} + \{\partial Q, \partial w_e\}_{d\dot{x}} \right] d\dot{x} \tag{6}$$

Equations (5) and (6) establish the fundamental relationship between *I, Q and P, V*.

3 Ultra-tight Kalman filter Architecture

The complementary Kalman filter structure for the ultra-tightly integrated system is shown in Figure 5

Fig. 5 Ultra-tight integration Kalman filter

As shown in Figure 5, the correlator measurements are processed with the INS predicted measurements in the integration Kalman filter to generate the inertial error estimates to correct the raw INS data. There are two update steps in the Kalman filter: *measurement* and *state update*. As the system's apriori knowledge is usually unknown at the start of the process, the mean of the state estimate x_{t0} and covariance matrix $P(0)$ is initialised to zero, i.e. $E[x_{t0}] = 0$ and $E[x_{t0}x_{t0}^T] = P(0)$ are assumed. $P(0)$ is a diagonal matrix corresponding to the '*error state variances*' of each state.

The dynamics of any linear time invariant system can be given as:

$$\dot{x} = Ax + \varepsilon \qquad \textbf{Process Model} \qquad (7)$$
$$z = Hx + v \qquad \textbf{Measurement Model} \qquad (8)$$

Where x is the state vector, A is the system matrix, H is the measurement matrix, z is the measurement vector, and ε and v refer to the process and measurement noises respectively. A 17-state Kalman filter is chosen for our studies.

$$x(t) = \{dx, dy, dz, d\dot{x}, d\dot{y}, d\dot{z}, \psi_x, \psi_y, \psi_z,$$
$$a_x, a_y, a_z, g_x, g_y, g_z, c_b, c_d\}^T$$
$$(9)$$

The 17 states are: 3 inertial error states each in position, velocity, attitude, accelerometer bias, gyro bias, 1 state each for clock bias and drift. The measurements presented to the complimentary filter are:

$$z = \{\text{INS predicted measurements}\} -$$
$$\{\text{GPS measurements}\}$$

$$z = \left\{dI + \eta_I, dQ + \eta_Q\right\}_{i=1:n}$$
$$(10)$$

$$z = \left\{I_{pred} + dI, Q_{pred} + dQ\right\}_{i=1:n} -$$
$$\left\{I_{GPS} - \eta_I, Q_{GPS} - \eta_Q\right\}_{i=1:n}$$
$$(11)$$

where dI, dQ are the deviations in the INS predicted I_{pred} and Q_{pred} measurements caused by the inertial sensor errors, and η_I, η_Q are the quadrature noise components in the GPS I_{GPS} and Q_{GPS} measurements respectively. Suffix 'i' in equation (11) represents the number of channels tracked. The predicted INS measurements are derived from the knowledge of the current INS position and velocity. The equations pertaining to this were introduced in the previous sections.

3.1 Process Model

To start the Kalman filter, the process and measurement models need to be defined. In equation (7), the dynamics matrix A is defined using terrestrial psi-angle error model (Bar-Itzhack, 1988; Wang et al., 2001):

$$d\dot{r} = -\rho * dr + dv$$
$$d\dot{v} = -(\Omega * w) * dv + \nabla - \psi * f$$
$$d\dot{\psi} = -w * \psi + \varepsilon$$
$$(12)$$

where dr is the position error vector, dv is the velocity error vector, $d\psi$ is the attitude error vector, ρ is the true frame rate with respect to the Earth, Ω is the Earth rate vector,

w is the true coordinate system angular rate with respect to the inertial frame, ∇ is the accelerometer error vector, f is the specific force vector, and ε is the gyro drift rate vector.

3.2 Measurement Model

The measurement model H is given as (Babu & Wang, 2005):

$$H = [\{h_{xi}, h_{yi}, h_{zi}, 1\}, \{h_{\dot{x}i}, h_{\dot{y}i}, h_{\dot{z}i}, 1\}]_{i=1:n}$$

(13)

where 'n' is the number of satellites visible and

$$h_{x1} = \left[\frac{\partial E[I]}{\partial \phi_e} \frac{\partial \phi_e}{\partial x} + \frac{\partial E[I]}{\partial w_e} \frac{\partial w_e}{\partial x} \right]$$

and

$$\dot{h}_{x1} = \left[\frac{\partial E[Q]}{\partial \phi_e} \frac{\partial \phi_e}{\partial \dot{x}} + \frac{\partial E[Q]}{\partial w_e} \frac{\partial w_e}{\partial \dot{x}} \right]$$

The H matrix is updated during the measurement update process. The product of the predicted states and the H matrix are then differenced with the GPS measurements, and weighted by the Kalman gain to generate the inertial error estimates.

4 Simulation Experiments

Simulation experiments were performed to test the performance of the filter. From a constant velocity trajectory of 100m/s, the GPS and INS measurements were extracted and fed into the Kalman filter. The update rate of the filter is 10Hz. Using a Matlab-based Software Receiver, the I and Q signals for 4 channels, PRN 4, 6, 7, 10, as shown in Figure 2, were derived. A INS Matlab toolbox generated the inertial sensor measurements. The U-D covariance factorisation algorithm was used to ensure the positive-definiteness of the covariance matrix P. To reduce the computational burden, the scalar update method was followed. The Kalman filter was run for 1000 seconds and the error estimates between the reference and the estimated trajectories are plotted in Figures 6 and 7.

Fig. 6 Attitude Error Estimates
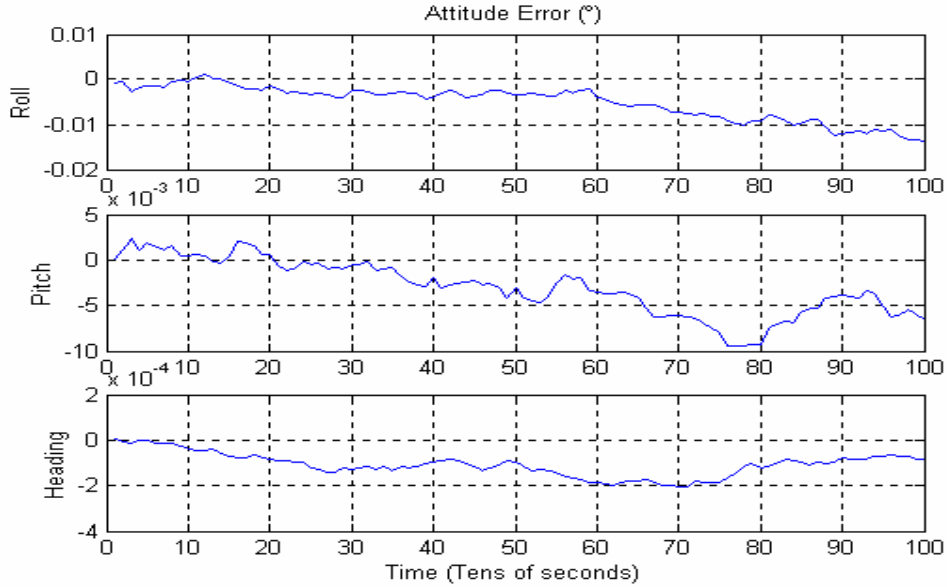
As Figure 6 shows, there is a bias in the position estimates which may be due to synchronisation and modelling errors. However, the bias is bounded, which is critical .

for the stability of the filter. The attitude errors also show consistency. Figures 8, 9 & 10 show the accuracies for the gyro bias, the accelerometer bias and position estimates.
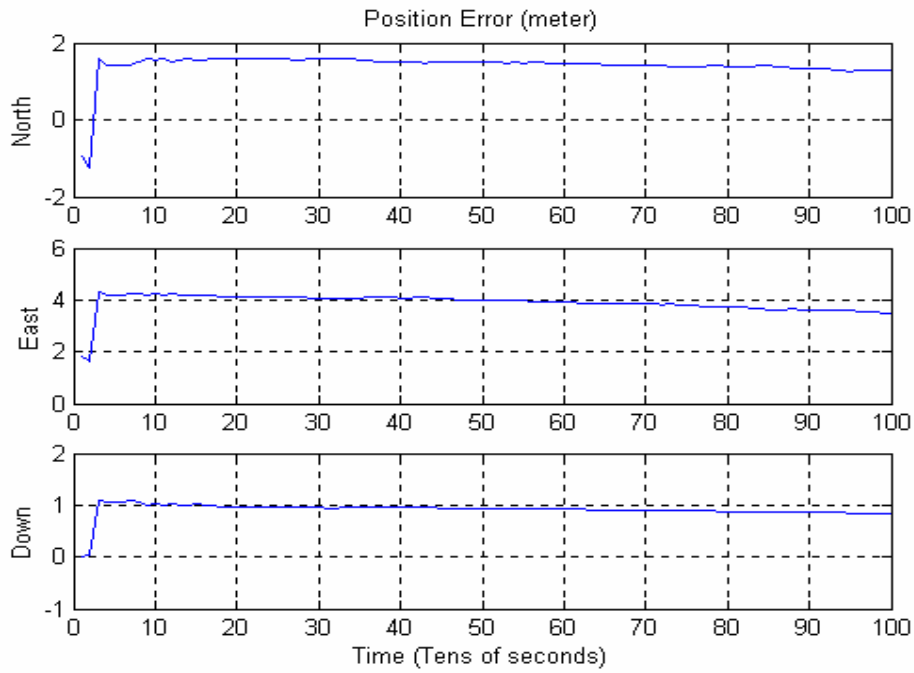
Fig. 7 Position Error Estimates

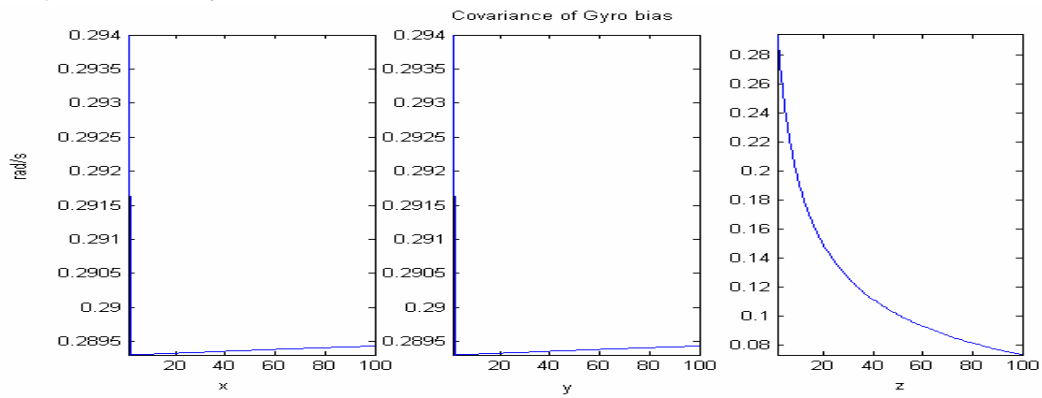**Fig.** 8 Covariance of Gyro bias

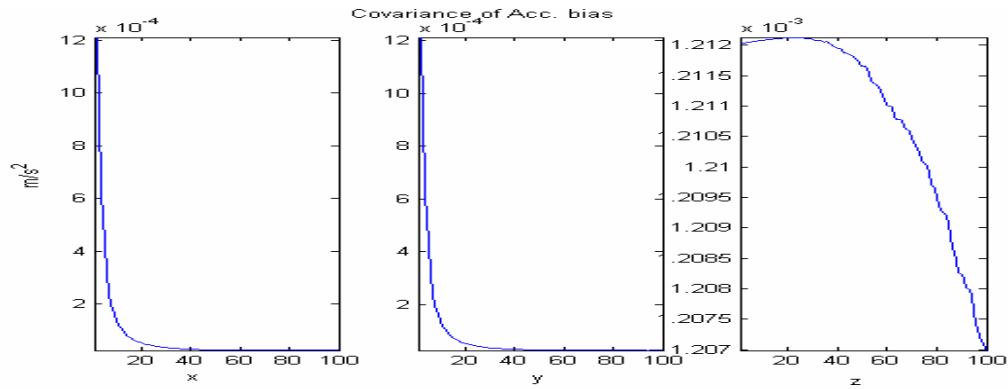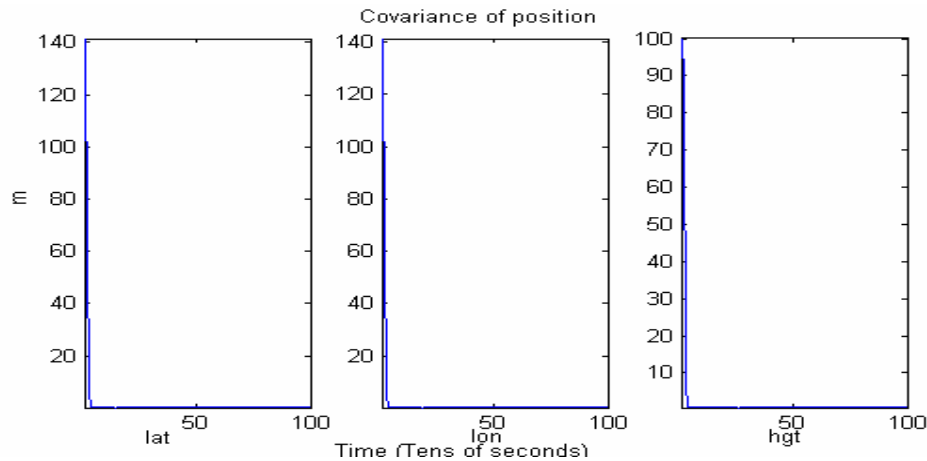**Fig. 9** Covariance of Acc. bias

Fig. 10 Covariance of position

5 Concluding Remarks

One of the complexities of an ultra-tight integration system is the Kalman filter implementation. Unlike loosely and tightly integrated systems, the mathematical relationships that relate the states and measurements are complex. The states and measurements are related through the phase and frequency errors extracted from the receiver tracking loops. This paper describes these relationships, which are critical to the understanding of the system. The tracking performance of a stand-alone and ultra-tight GPS receiver illustrates the advantages of the ultra-tight system under dynamic conditions. It is shown that the ultra-tight integration receiver with a carrier tracking bandwidth of 3Hz operates in the PLL mode even under high dynamic conditions. The error estimates and the covariance analysis have shown that the system's performance is encouraging. However, these are only preliminary results and optimisation is still required in the modelling of the various parameters of the filter. With rapidly evolving low-cost inertial sensors and low-cost GPS receivers, ultra-tightly integrated systems may be poised to capture commercial and defence markets.

Acknowledgments

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